

arccos, atan2 from trigonometry

ElementSets from MeshConnectivity

Faces, VertexOneRing, OrientedOppositeFaces, OppositeVertices, NeighborVerticesInFace, OrientedVertices from Neighborhoods(M)

$M : \text{FaceMesh}$

$x_i \in \mathbb{R}^3$

$V, E, F = \text{ElementSets}(M)$

$\text{VertexNormal}(i) = \left(\sum_{j \in \text{Face}(i)} \frac{(x_j - x_i) \times (x_k - x_i)}{\|x_j - x_i\|^2 \|x_k - x_i\|^2} \right) \text{ where } j, k = \text{NeighborVerticesInFace}(f, i) \text{ where } i \in V$

$\theta(i, f) = \arccos \left(\frac{(x_j - x_i) \cdot (x_k - x_i)}{\|x_j - x_i\| \|x_k - x_i\|} \right)$

where

$i \in V$

$f \in F$

$j, k = \text{NeighborVerticesInFace}(f, i)$

$\text{area}(f) = \frac{1}{2} \|(x_j - x_i) \times (x_k - x_i)\|$

where

$f \in F$

$i, j, k = \text{OrientedVertices}(f)$

$N(f) = \frac{(x_j - x_i) \times (x_k - x_i)}{2 \text{area}(f)}$

where

$f \in F$

$i, j, k = \text{OrientedVertices}(f)$

$l(i, j) = \|x_j - x_i\| \text{ where } i, j \in V$

$\phi(i, j) = \text{atan2}(e \cdot (n_1 \times n_2), n_1 \cdot n_2)$

where

$i, j \in V$

$e = \frac{x_j - x_i}{\|x_j - x_i\|}$

$f_1, f_2 = \text{OrientedOppositeFaces}(i, j)$

$n_1 = N(f_1)$

$n_2 = N(f_2)$

$\cot(k, j, i) = \frac{\cos}{\sin}$

where

$i, j, k \in V$

$of, ot = \text{OrientedVertices}(k, j, i)$

$\cos = (x_{of} - x_k) \cdot (x_{ot} - x_k)$

$\sin = \|(x_{of} - x_k) \times (x_{ot} - x_k)\|$

$KN(i) = \frac{1}{2} \left(\sum_{j \in \text{VertexOneRing}(i)} \frac{\phi_{k,i}}{l_{i,j}} (x_j - x_i) \right) \text{ where } i \in V$

$HN(i) = \frac{1}{2} \left(\sum_{j \in \text{VertexOneRing}(i)} (\cot(\alpha) + \cot(\beta))(x_i - x_j) \text{ where } k, p = \text{OppositeVertices}(i, j), \cot(\alpha) = \cot(k, j, i), \cot(\beta) = \cot(p, i, j) \right) \text{ where } i \in V$